

1 **Comment on “Heat capacity, time constant, and**  
2 **sensitivity of Earth’s climate system” by Schwartz.**

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4 Schwartz [2007] recently evaluated the time constant of Earth’s climate system. How-  
5 ever, his methodology yields to a significant underestimation of the value of  $\tau$  and obscures  
6 a much more interesting property of the system. Schwartz found  $\tau = 5 \pm 1a$ . Herein, by  
7 using an improved methodology I find that for short time scales from 0 to 2 years  $\tau$  is of  
8 the order of a several months and for larger time scales, at least up to 20 years,  $\tau$  is at  
9 least 70% larger than what Schwartz estimated.

10 Schwartz [2007] hypothesized (his Eq. 17) that the climate system behaves as a first-  
11 order autoregressive process plus a linear trend. The implicit idea seems that the linear  
12 trend represents the effect of the external forcings on climate while the temperature sig-  
13 nal, detrended of the above linear component, represents the internal variability of the  
14 same. This internal variability is assumed to be described by an AR(1) process whose

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15 autocorrelation function,  $r(\Delta t)$ , decays as an exponential function of the lag-time  $\Delta t$  with  
 16 a given time constant  $\tau$ :  $r(\Delta t) = \exp(-\Delta t/\tau)$ .

17 Although in physics using simple models is useful, the one suggested by Schwartz, with  
 18 a single time constant, is an oversimplification and, as I will prove below, it is inconsistent  
 19 with the analysis. In fact, it is very well known that climate is the combination, cou-  
 20 pling and superposition of several phenomena. Some phenomena respond quickly as the  
 21 atmosphere, others as the deep ocean respond very slowly. Thus, each climate component  
 22 responds with its own time constant that might range from a few months to several years  
 23 or decades.

Given the length limitation of the temperature data herein analyzed (approximately 125 years) the analysis is limited to time scales below 20 years with a monthly resolution and I look for two time constants. The climate model I suggest is

$$R_t = T_t - F_t = X_t + Y_t, \quad (1)$$

where  $t = 1, 2, \dots$  is a discrete time index and

$$X_t = a_1 X_{t-1} + \zeta_t \quad (2)$$

$$Y_t = a_2 Y_{t-1} + \eta_t. \quad (3)$$

So, we have that  $T_t$  represents the global temperature,  $F_t$  is the climate effect of the external forcings,  $R_t$  the residual signal. The residual signal is made of a slow plus a fast AR(1) processes,  $X_t$  and  $Y_t$ , respectively, which describe the internal variability of climate.  $\zeta_t$  and  $\eta_t$  are two independent white noise processes with zero mean and standard deviation  $\sigma_1$  and  $\sigma_2$ , respectively. The above model has the residual signal  $R_t$  characterized by the

following autocorrelation function

$$r(\Delta t) = A_1 \exp(-\Delta t/\tau_1) + A_2 \exp(-\Delta t/\tau_2), \quad (4)$$

where  $A_1 + A_2 = 1$ . By calling  $a_1 = \exp(-\Delta/\tau_1)$ ,  $a_2 = \exp(-\Delta/\tau_2)$ , where in our case  $\Delta = 1/12$  a, we have

$$\frac{\sigma_1^2}{\sigma_2^2} = \frac{A_1(1 - a_1^2)}{A_2(1 - a_2^2)}, \quad (5)$$

thus the relative magnitude between  $A_1$  and  $A_2$  is directly proportional to the relative magnitude of the two noise variances.

Fig. 1A shows a global surface temperature record from 1880 to 2008 [Brohan *et al.*, 2006] (note that the record includes data from 1850 to 1880 too, but they are excluded herein because Schwartz excluded them). Fig. 1B shows the temperature record detrended of the linear component, as Schwartz does. The autocorrelation function of a time series  $\{\xi_i\}$  with  $i = 1, 2, \dots, N$  (mean  $\mu$  and variance  $\sigma^2$ ) is

$$r(\Delta) = \frac{1}{(N - \Delta)\sigma^2} \sum_{i=1}^{N-\Delta} (\xi_i - \mu)(\xi_{i+\Delta} - \mu), \quad (6)$$

where  $\Delta$  is the lag-time. Fig. 1C shows  $r(\Delta t)$  of the sequence plotted in Figure 1B and its fit with the above autocorrelation function within the interval from 0 to 11 years where the results are more stable. I obtain  $\tau_1 = 0.40 \pm 0.1$  a in the short range and  $\tau_2 = 8.7 \pm 2$  a in the long range, respectively. The figure also shows Schwartz's equation with  $\tau = 5$  a and it is evident that it does not fit the data. Note that in his Figures 6 and 7, Schwartz claims that  $\tau(\Delta t)$  monotonically increases in  $\Delta t$  from 0 to 5 a. But, as Figure 1 shows, this is not correct. Indeed, Schwartz's result is an mathematical artifact of using a single AR(1) process. Also, I find  $\sigma_1/\sigma_2 \approx 5$ .

34 However, the above conclusion follows by assuming, as Schwartz did, that the internal  
35 variability of climate can be deduced by simply removing a linear trend from the temper-  
36 ature data. This choice is evidently questionable. I repeat the calculation by removing  
37 from the temperature data the average global temperature GISS simulation obtained by  
38 using all forcings [Hansen *et al.*, 2007]. The rationale is that the residual shown in Fig. 2B  
39 represents the internal variability of the climate system as obtained by the GISS model.  
40 Fig. 2C shows the result. I obtain  $\tau_1 = 0.39 \pm 0.1$  a in the short range and  $\tau_2 = 8.1 \pm 2$   
41 a in the long range, respectively. These values do not differ significantly from the pre-  
42 vious ones obtained with a simpler linear detrending, and suggest that the above results  
43 might be quite robust, unless the GISS model is found to be extremely poor. Also, I find  
44  $\sigma_1/\sigma_2 \approx 6$ .

45 However, the length of the time series herein analyzed is quite short, and the time  
46 constants might be underestimated. To estimate the magnitude of the statistical bias  
47 I compare the autocorrelation function of very long computer generated time series ac-  
48 cording to the above model with sequences of  $125 \times 12 = 1500$  data, as the sequences herein  
49 analyzed. The results is shown in Fig. 3. It seems that  $\tau_1$  is not significantly changed, the  
50 real  $\tau_2$  might be 50% larger than the measured one, and the real  $A$  might be 10% smaller  
51 than the measured one.

52 Thus, the figures show that within a time scale of one-two years the climate is char-  
53 acterized by a fast time response of about 5 months while for time scales larger than  
54 one-two years up to 20 years the climate system is characterized by a slower response  
55 with a measured time constant of about  $8 \pm 2$  a, which may correspond to  $12 \pm 3$  a by

56 taking into account the statistical bias. These estimates are significantly larger than what  
57 Schwartz calculated, but well agree with what found in Scafetta and West [2007] with  
58 an alternative model by adopting the latest solar and temperature proxy sequences since  
59 1600:  $\tau = 9 \pm 3.25$  a.

60 By trusting Schwartz's equations about the equilibrium climate sensitivity,  $\lambda_s^{-1} = \tau/C$ ,  
61 and assuming that  $\tau = \tau_2$ , I obtain a value that ranges from a measured  $\lambda_s^{-1} =$   
62  $0.5K/Wm^{-2}$  to a hypothetical  $\lambda_s^{-1} = 0.7K/Wm^{-2}$ . These values are below but com-  
63 patible with the estimates summarized in the Fourth Assessment Report of the IPCC  
64 [2007]:  $\lambda_s^{-1} = 0.8_{-0.3}^{+0.4}K/Wm^{-2}$ . The above values correspond to an equilibrium tempera-  
65 ture increase for doubled  $CO_2$ ,  $\Delta T_{2X}$ , from a measured best estimate of about 1.7 K to a  
66 hypothetical one of about 2.6 K: the IPCC best estimate is about 3 K, from a minimum  
67 of 1.5 K to a maximum of 4.5 K. However, the value of  $\tau$  required in the above equi-  
68 librium climate sensitivity might be larger than  $\tau_2$  because it might refer to a secular or  
69 millenarian time scale, while  $\tau_2$  refers to a decadal time scale.

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72 **Figure 1:** [A] Global monthly average surface temperature [*Brohan et al.*, 2006]. [B]  
73 Detrended sequence. [C] Autocorrelation function of the detrended sequence show in [B].  
74 The y-axis is in logarithmic scale.

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76 **Figure 2:** As in Figure 1 but instead of a linear trend I detrend the average global  
77 temperature GISS modelE simulation (monthly moving average) obtained by using all  
78 forcings [*Hansen et al.*, 2007].

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80 **Figure 3:** Comparison between autocorrelation functions for computer generated data.  
81 A result with a long sequence obtained with  $A_1 = 0.65$ ,  $\tau_1 = 0.4$  a,  $\tau_2 = 12$  a is compared  
82 with an average result obtained with ten sequences of  $125 \times 12 = 1500$  data. The curves are  
83 fit with the function:  $r(\Delta t, A_1, \tau_1, \tau_2)$ .

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